*J. Fluid Mech.* (1985), *vol.* 166, *pp.* 531–534
*Printed in Great Britain*

# REVIEWS

**Nonlinear Dynamics and Turbulence.** Edited by G. I. BARENBLATT, G. IOOSS & D. D. JOSEPH. Pitman, 1983. 356 pp. £35.00.

For fluid dynamicists, interest in the connection between 'turbulence' and the behaviour of systems of ordinary differential equations stems from the pioneering work of Lorenz. He showed that relatively simple *deterministic* systems of equations modelling fluid-dynamical phenomena can exhibit solutions whose details are unpredictable. Since that time there has been a burgeoning of activity by both pure and applied mathematicians, resulting in much useful cross-fertilization. On the one hand, theoreticians have mapped out several possible ways in which irregular behaviour can arise in dynamical systems as a result of sequences of bifurcations. On the other, a new generation of experimentalists, many with a background in solid-state physics, has successfully devised experiments involving fluid motions of very simple spatial structure, so that the dynamics may be accurately represented by the interaction of a small number of terms in a Galerkin expansion. It has thus been possible to predict the steps in the transition from regular to irregular flow in these simple systems by direct analogy with theory. There is no doubt that our understanding of such systems has been greatly enhanced, but it is not at all clear whether the phenomena being described bear any relation to turbulent shear flow, wake flow or similar manifestations of disorder in systems possessing *many* degrees of freedom.

The last point is admitted by the editors of the present volume, who have worked on aspects of dynamics and stability theory in the USSR, France and the USA respectively. Their laudable aim has been to bring together a series of articles from these three countries giving an overview of current research. The Russian contribution is particularly welcome since it is less widely known in the West. It has to be said, however, that only a minority of the articles deal with the dynamics of fluids directly. There is a good short review by Gollub of recent experimental results on the transition to turbulence in convection at both small and large aspect ratio. Particularly interesting is his discussion of the irregular motion observed near the onset of convection in large boxes, where the theory for an infinite layer predicts a stable time-independent roll pattern. Busse contributes an article on the nonlinear evolution of the Kuppers–Lortz instability of convection in a rotating layer. This is important as an example of irregular motion just above threshold, which can be adequately described by only three coupled o.d.e.'s. A curiosity here is that the system is not structurally stable (a fact not remarked on by Busse) but the addition of higher-order terms leads to qualitative behaviour different from that observed in experiments, if the stochastic terms (essential to Busse's analysis) are ignored. As a general rule it is important, when attempting to resolve degeneracies in evolution equations by including small terms, to consider whether external noise is more important in determining the dynamical behaviour.

Russian work on the Taylor–Couette problem is represented by a long and interesting paper by L'vov, Predtchensky & Chernykh. It describes experiments (principally) on the wavy instability of Taylor cells, and their transition to turbulence. These parallel the work of Swinney and his collaborators, but in somewhat different parameter ranges. The experiments are compared, not with any set of 'normal-form equations' derived from centre-manifold theory, but with a set of 15 equations (one for each of the vortices in the experiment) with interaction terms chosen to give a simple representation of the physics. This model is quite useful in predicting qualitative features of the transition to turbulence, although recent work by Golubitsky, Stewart and others on bifurcation with symmetry suggests that more accurate representation of the spatial, as distinct from the temporal structure might be possible. The paper also includes a discussion of stochastic theories of turbulence and the Direct Interaction approximation as well as investigating the dimension of the attractor (the set of points in phase space towards which trajectories tend) in the turbulent regime. The last question is an important one in the understanding of turbulence in general but although there is a theoretical foundation to be built on little progress has been made to date with real flows.

Among the more mathematical articles is one on the ability of the usual Galerkin approximation used in numerical modelling of turbulent flows to reproduce the dynamics of the continuous system (Foias & Temam). We all take such ability for granted, but unfortunately a full theoretical justification seems difficult except for steady or simple oscillatory flows (though more can be done in two dimensions than in three). Treve has contributed a short paper on the selection of Galerkin schemes that satisfy certain energy integrals obeyed by the full problem. If the correct choice is made one can prove boundedness and other useful properties of the solutions. The great majority of the remaining papers are 'pure mathematics': they range from a clear review by Feigenbaum of his own seminal work on period-doubling bifurcations of the logistic equation (shown more recently to be an important ingredient in the transition to chaos in many simple systems), through several papers on bifurcation theory, down (up?) to a paper on 'Strange attractors and quasi attractors' by Afraimovich & Shil'nikov whose mathematical style made it incomprehensible to this reviewer.

In summary, this is a collection of articles of more than ephemeral interest, though few present important new work. I would think it a useful addition to an Applied-Mathematics library, though there is little in it for those on the engineering side of turbulence theory.

MICHAEL PROCTOR

**Large-Scale Dynamical Processes in the Atmosphere.** Edited by B. J. HOSKINS and R. P. PEARCE. Academic, 1983. 390 pp. £33.00.

Dynamical meteorology is entering an exciting time. 'Particularly notable in the last few years', to quote from the editors' introduction to this volume, 'has been the way that the theoreticians and those whose strengths have been in manipulating observational data have intensified their collaboration in a determined attempt to understand atmospheric behaviour.' This trend continues today, and is in tune with worldwide concerns over the response of our atmosphere and oceans to disturbances whether natural or man-made. Large amounts of data have become available from a new generation of ingenious terrestrial and space-based observing systems. These have revealed fascinating new views of our large-scale fluid-dynamical environment and have led to important new theoretical insights. Theoretical ideas, including some which once seemed remote and academic, are for their part leading to new and more powerful ways of making sense of the massive amounts of data involved.

This book well exemplifies today's renewed efforts to build bridges between observation and theory. It is an important reference for anyone interested in the large-scale dynamics of naturally occurring bodies of fluid. Although there is only one chapter devoted to oceanic circulations, and none to the atmospheres of planets other than the Earth, our own atmosphere, which forms the main topic of this book, is by far the most intensively observed geophysical fluid system. The book emerged from an international symposium held in August 1981 at the University of Reading, England, attended by a large number of leading atmospheric researchers. It consists of a collection of in-depth review articles by distinguished experts on observational data and on dynamical theory, and is far more than a mere conference proceedings. The authors and editors have made strenuous efforts to provide uniformity of notation, and a considerable amount of cross-referencing and indexing, and additional material, so that 'the book may be read as an advanced text on large-scale atmospheric dynamics'. Quasi-geostrophic theory inevitably plays an important conceptual role, and there is a brief but useful appendix summarizing the main points of that theory, including the (almost but not quite straightforward) approximate relation between quasi-geostrophic potential vorticity and Rossby–Ertel potential vorticity. For some reason good explanations of this point seem difficult to find in the literature beyond the seminal 1962 paper by Charney & Stern.

Chapters written by M. L. Blackmon, R. M. Dole, E. O. Holopainen, I. N. James, N.-C. Lau, and J. M. Wallace survey the recent intensive work on the global-scale spatio–temporal patterns which emerge from long and short time series of extratropical tropospheric data from operational meteorological networks, and from large numerical models, filtered so as to bring out the behaviour on timescales ranging from days to months. There is a healthy trend towards flexibility in trying different ways of viewing the data suggested by different theoretical viewpoints. Some of the latter are expertly presented in theoretically oriented chapters by I. M. Held, B. J. Hoskins, and C. E. Leith, and a provocative essay on both the observational and the theoretical aspects of the sparsely observed Tropics is contributed by P. J. Webster. Here the effect of the ocean is especially important, and may be a key to long-term predictability. The chapter on the ocean circulation itself, with special reference to its coupling to the atmosphere, is contributed by D. L. T. Anderson; and L. Bengtsson and A. J. Simmons give an authoritative account of one of the key practical tests of our grasp of atmospheric processes and our ability to make use of heterogeneous data sets, the problem of medium-range weather prediction.

The quasi-biennial oscillation (QBO) of the zonal winds in the equatorial lower stratosphere, at altitudes between 15 and 35 km, remains the only atmospheric phenomenon, apart from seasonal changes, which can be predicted with confidence as far as several years ahead. It is also a phenomenon which has never been successfully simulated, even qualitatively, by the largest and most expensive numerical models of the atmosphere, the kind which represent our best attempts to capture as much of the atmosphere's behaviour as possible for the purpose of detailed weather forecasting or of simulating climatologically relevant statistics. The discovery of its likely mechanism, which is a quintessentially fluid-dynamical mechanism, involving a geophysical analogue of acoustic streaming, is a fascinating scientific detective story. Our present understanding is reviewed by one of its pioneers, J. R. Holton, as part of the remaining chapter, which discusses the stratosphere and its links to the troposphere. The subtlety of the QBO's likely mechanism is not the only dynamical lesson we have learned from the stratosphere. There have recently been others which now promise insights of an entirely unforeseen kind into phenomena, such as extratropical cyclogenesis, previously thought of as distinctively tropospheric. Some of these new insights are discussed in the theoretical chapters by Held and Hoskins.

Fluid dynamicists will be interested in the various species and genera of coherent structures that are discussed in this book. These are of at least two kinds. The first is seen mainly in the form of statistically generated patterns from long time series of data (the longest being a data set for 18 winters). There are intriguing hints that linear processes like Rossby-wave propagation and dispersion help to shape some of these patterns. Alternative ideas involving large-scale instabilities have also been suggested, and these may yet turn out to be closely related since the instabilities in question all involve the Rossby-wave propagation mechanism in one way or another. Some for instance may be able to be thought of as involving local (but leaky) 'cavities' which tune themselves towards resonance as part of the instability mechanism. A definitive interpretation of the observed patterns has yet to be reached. This reviewer thinks that some of them will turn out to involve Rossby-wave *breaking* as well as Rossby-wave propagation.

A second but not unrelated kind of coherent structure is that of individual dynamical episodes and sequences of episodes such as the persistent 'blocking anticylones' which brought drought to the UK in the summers of 1976 and 1983. No-one doubts that the individual structures and the long-term statistical patterns are related in some way, but there is a chicken–egg problem and the causal links still present unresolved puzzles. (Rossby-wave breaking may be relevant here too.) The chapter by Leith presents a provocative personal view suggesting that some of the individual structures should be thought of in partial analogy to solitons and other quasi-permanent dynamical structures studied by mathematicians and quantum field theorists. Whatever the imperfections of such analogies, they are certainly stimulating to thinkers interested in the profoundly difficult problem of variations in atmospheric predictability and the persistence or otherwise of different weather patterns.

Does the book live up to its stated aim of providing an advanced text? I suppose the answer depends on how advanced. There is a great deal of material to stimulate the researcher here, and an attempt to let the ideas and insights of imaginative individuals come through freshly and strongly. I would have been inclined to put the book somewhat more into the category of a research monograph, or perhaps one should say polygraph. It is certainly a valuable complement to the well-known, more formal texts of Gill, Holton and Pedlosky. I would enthusiastically recommend it to all researchers and graduate students interested in the atmosphere.

M. E. McINTYRE